SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

Does $\lim_{n\to\infty} a_n = 0$? —— NO —— $\sum a_n$ **Diverges**

YES

p-SERIES

Does $a_n = 1/n^p$, $n \geq 1$? —— YES —— Is $p > 1$?
- YES —— $\sum a_n$ **Converges**
- NO —— $\sum a_n$ **Diverges**

NO

GEOMETRIC SERIES

Does $a_n = ar^{n-1}$, $n \geq 1$? —— YES —— Is $|r| < 1$?
- YES —— $\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$
- NO —— $\sum a_n$ **Diverges**

NO

ALTERNATING SERIES

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n$, $b_n \geq 0$? —— YES —— Is $b_{n+1} \leq b_n$ & $\lim_{n\to\infty} b_n = 0$? —— YES —— $\sum a_n$ **Converges**

NO

TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form. —— YES —— Does $\lim_{n\to\infty} s_n = s$ s finite?
- YES —— $\sum a_n = s$
- NO —— $\sum a_n$ **Diverges**

NO

TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!}(x-a)^n$? —— YES —— Is x in interval of convergence?
- YES —— $\sum_{n=0}^{\infty} a_n = f(x)$
- NO —— $\sum a_n$ **Diverges**

NO

Try one or more of the following tests:

COMPARISON TEST

Pick $\{b_n\}$. Does $\sum b_n$ converge? —— YES —— Is $0 \leq a_n \leq b_n$? —— YES —— $\sum a_n$ **Converges**

NO —— Is $0 \leq b_n \leq a_n$? —— YES —— $\sum a_n$ **Diverges**

NO

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ c finite & $a_n, b_n > 0$? —— YES —— Does $\sum_{n=1}^{\infty} b_n$ converge?
- YES —— $\sum a_n$ **Converges**
- NO —— $\sum a_n$ **Diverges**

INTEGRAL TEST

Does $a_n = f(n)$, $f(x)$ is continuous, positive & decreasing on $[a, \infty)$? —— YES —— Does $\int_a^{\infty} f(x)dx$ converge?
- YES —— $\sum_{n=a}^{\infty} a_n$ **Converges**
- NO —— $\sum a_n$ **Diverges**

RATIO TEST

Is $\lim_{n\to\infty} |a_{n+1}/a_n| \neq 1$? —— YES —— Is $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$?
- YES —— $\sum a_n$ **Abs. Conv.**
- NO —— $\sum a_n$ **Diverges**

ROOT TEST

Is $\lim_{n\to\infty} \sqrt[n]{|a_n|} \neq 1$? —— YES —— Is $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$?
- YES —— $\sum a_n$ **Abs. Conv.**
- NO —— $\sum a_n$ **Diverges**